

## Price Discrimination in Quantity Setting Oligopoly

Kumar, R., & Kutlu, L. (2016). Price Discrimination in Quantity Setting Oligopoly. *Manchester School*, 84(4), 482-505. <https://doi.org/10.1111/manc.12106>

**Published in:**  
Manchester School

**Document Version:**  
Peer reviewed version

**Queen's University Belfast - Research Portal:**  
[Link to publication record in Queen's University Belfast Research Portal](#)

### **Publisher rights**

© 2015 The University of Manchester and John Wiley & Sons Ltd.

This is the accepted version of the following article: Price Discrimination in Quantity Setting Oligopoly, which has been published in final form at DOI: 10.1111/manc.12106

### **General rights**

Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

### **Take down policy**

The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact [openaccess@qub.ac.uk](mailto:openaccess@qub.ac.uk).

# Price Discrimination in Quantity Setting Oligopoly

Rajnish Kumar

Queen's University Management School

Queen's University Belfast

Levent Kutlu

School of Economics

Georgia Institute of Technology

Revised, January 18, 2015.

Keywords: Price discrimination; Oligopoly; Cournot competition

JEL Codes: L11, L13

## Abstract

We analyze a two-stage quantity setting oligopolistic price discrimination game. In the first stage firms choose capacities and in the second stage they simultaneously choose the share that they assign to each segment. At the equilibrium the firms focus more on the high-valuation customers. When the capacities in the first stage are endogenous, the deadweight loss does not vanish with the level of price discrimination, as it does in one-stage games and monopoly. Moreover, the quantity-weighted average price increases with the level of price discrimination as opposed to established results in the literature for one-stage games.

## 1 Introduction

While the theory of price discrimination is well understood in the monopoly setting, this literature is relatively new for imperfectly competitive markets. There are two broad approaches to modelling imperfect competition: price competition and quantity competition. The differentiated products framework is dominant in the price competition setting and the homogeneous goods framework is dominant in the quantity competition setting.<sup>1</sup> The common approach in the literature on

---

<sup>1</sup>See Armstrong [1], Renault and Anderson [14], and Stole [19] for surveys about price discrimination.

price discrimination in oligopoly follows the price competition approach. Borenstein [3], Chen [4], Holmes [8],<sup>2</sup> and Thisse and Vives [21] exemplify some of the studies that use the price competition approach.

In the present paper we concentrate on the homogeneous goods framework where the valuations of customers are represented by a function of some characteristic(s) of the customers. On the basis of those characteristics firms segment the customers in various segments. We analyze price discrimination as a two-stage game. In the first stage the firms compete on quantities that they put in the market, and in the second stage they simultaneously decide what fraction of the quantity they sell to different groups of buyers.<sup>3</sup> In other words, in the second stage firms optimally segment their customers by allocating the available quantities. Hence, in the second stage the firms can be asymmetric if their first stage choices are not the same. Firms may have many instruments at their disposal for discriminating between buyers. Where our model can be applied, the examples abound. A prominent example of an instrument used for price discrimination is the airline industry, where the valuations of buyers can be represented by a function of the time when they buy tickets. Business travellers, whose plans are generally last moment, are willing to pay more compared to tourists, whose plans are almost always flexible. Thus different segments of buyers can be grouped according to the day they want to buy a particular airline seat. For example, higher-end segments may consist of business travellers. In order to fix ideas we stick to the airline industry.

The paper closest to ours that considers price discrimination in quantity competition is Hazledine [6]. For the linear demand case, Hazledine [6] analyzes price discrimination in the Cournot framework, where firms decide on quantities to sell in various segments. In contrast to Hazledine [6], who models price discrimination in one stage, we model price discrimination in two stages, as described earlier. In both Hazledine [6] and our model, the number of segments is exogenous. Hazledine [6] finds that the average price in the market is independent of the level of price discrimination and thus concludes that the standard single-price models' prediction is not misleading in terms of the average price. Bakó and Kálec-Simon [2] and Kutlu [11] confirm the robustness of invariance of average

---

<sup>2</sup>Holmes [8] builds upon the monopoly model of Robinson [15].

<sup>3</sup>In another framework, Kreps and Scheinkman [9] propose a two-stage game where in the first stage the firms set the capacities and in the second stage the price is determined in a Bertrand-like competition given the capacities. It turns out that the capacities correspond to the Cournot output levels and the firms set Cournot prices in the second stage.

price to the level of price discrimination. For the linear demand case, Kutlu [10] incorporates price discrimination in the Stackelberg [18] model and finds that the leader does not price discriminate. All these models consider that the firms segment customers into different segments according to certain characteristics determining their valuation. Varian [23] provides an earlier example of the similar approach for price discrimination in monopoly. In quantity setting price discrimination for a monopoly Varian [23] finds that increase in output is necessary for price discrimination to be welfare increasing. Formby and Millner [5] consider the relationship between price discrimination and competition.<sup>4</sup> More precisely, they compare the social welfare of price discriminating monopoly (in Hazledine’s [6] framework) and that of single price Cournot competition. They find that when the demand curve<sup>5</sup> is concave (convex, linear), price discrimination with  $n$  prices produces greater (lesser, equal) output and welfare than a Cournot oligopoly with  $n$  competitors.

We consider a general demand function (not restricted to linear) for the homogeneous goods in a duopoly setting. We start with the second stage of the game where, for a given quantity from the first stage, we provide an algorithm to find the shares to be allocated in different segments. One of the findings of our paper is that in the second stage both firms are active in the higher-end segments and their allocation of quantities are the same for each higher-end segment until the smaller firm runs out its first-stage quantity. Unless the demand function is “too convex,” the shares of the higher-end segments are greater than those of the lower-end ones. Then, we analyze the first stage of the competition where firms choose quantities. We provide an explicit solution for linear demand as an illustration. We show firms’ behavior in the benchmark Cournot case. The total quantities sold by the Cournot oligopolist and total welfare increase with the level of price discrimination. We find that the deadweight loss always exists, no matter what the level of price discrimination is. This contrasts with the one-stage price discrimination game (Hazledine [6]) and monopoly. In both the one-stage game and monopoly, deadweight loss vanishes as the number of price segments grows. Therefore, our model serves as an example of the fact that competition may not always be welfare increasing. Indeed, in this case, allowing a monopolist to operate and redistributing an optimal tax collected from the monopolist could make everyone better off compared to the two-stage price discriminating duopoly. In particular, an antitrust authority valuing total welfare may have higher incen-

---

<sup>4</sup>Formby and Millner [5] call it Stackelberg price discrimination.

<sup>5</sup>Whenever we mention “demand curve” we mean “inverse demand curve.”

tives to approve mergers for duopolists in the airline industry compared to other industries where price discrimination is not practiced. That is, the required efficiency gain for welfare in the price discrimination framework is potentially lower.<sup>6</sup> Moreover, in contrast to the established results in the literature that we discussed above, we find that the average price is increasing with the level of price discrimination. Hence, the standard single-price models' prediction about the average price can potentially be misleading when firms first choose capacities and then allocate these capacities in a second stage. Therefore, caution should be taken when using the average price for estimation in empirical models.

In section 2, we introduce our model and present results for the share allocation for a general demand function. In section 3, we provide the solution of the share allocation game for the linear demand case. We also provide an extension of the Cournot game and compare our results with that of Hazledine [6]. Section 5 concludes, and we gather the major proofs in section 6 (Appendix).

## 2 The Model and Share Allocations

In this section we consider an oligopolistic competition model in which firms choose the sizes of segments optimally.<sup>7</sup> We provide a solution algorithm for a general demand function which we use when deriving our results for the linear demand case in the following sections. We rule out the arbitrage possibilities, including intra-personal arbitrage, where a high-valuation customer can act like a low-valuation customer. Hence, in this sense, our model is a version of third-degree price discrimination (group pricing). The price in a segment is determined through the quantity choices of firms and depends on how much is sold in other segments. For example, consider a market with two segments based on the reservation prices, which depend on the number of days before the flight where there is a breakdown in prices. Airlines can choose quantities assigned to each segment indirectly by choosing a threshold day so that the customers arriving later than this day are assigned to segment 1 and the remaining customers are assigned to segment 2. By changing the threshold day, the airline can implicitly decrease the size of segment

---

<sup>6</sup>Note, however, that such a redistribution can be difficult to implement in practice. Also, merger decisions would likely be based on a variety of factors, including but not limited to firms' price discrimination behaviors. Here, we point out that the presence of price discrimination may not necessarily lead to welfare-decreasing outcomes.

<sup>7</sup>Note that the number of segments is exogenous. By optimal segmentation we mean the distribution of quantities in various segments rather than the number of segments, which is exogenous in our model.

1. Hence, whenever such adjustments are difficult or impossible to implement, the model with exogenous segmentation discussed above seems more sensible. For example, if the segmentation is based on gender, the size of segments would be taken as given. On the other hand, if the airlines are flexible in such adjustments, then our model may be more sensible. It seems that price discrimination based on advance purchase segmentation as described above allows the airlines to be flexible in adjusting the size of segments and thus our model fits this framework well.

Now we describe our theoretical model which is based on Hazledine [6]. Assume for simplicity that there are only two firms in the market that sell a homogeneous good: Firm  $A$  and Firm  $B$ . We set the marginal costs equal to zero. Each consumer buys at most one unit of the good and they buy the good only if the price does not exceed their valuation of the good. The firms know the valuations of consumers and can prevent resale of the good. They divide the consumers into  $K$  segments according to their reservation prices. The total capacities of the firms are exogenously given by  $Q_A$  and  $Q_B$  where  $Q_A \leq Q_B$ . Here we assume that the capacities are not too big so that disposing is not optimal. For example, we can consider  $Q_A$  and  $Q_B$  as quantities produced in the first stage of a two-stage game where production takes place in the first stage and the decision of quantity allocation in various segments is made in the second stage. Given these capacities firms compete on the shares that they assign to each segment. Hence, firms choose  $s_A = (s_A^1, s_A^2, \dots, s_A^{K-1}, s_A^K)$  and  $s_B = (s_B^1, s_B^2, \dots, s_B^{K-1}, s_B^K)$  with  $\sum_{i=1}^K s_A^i = 1$  and  $\sum_{i=1}^K s_B^i = 1$  where  $q_A^i = Q_A s_A^i$  and  $q_B^i = Q_B s_B^i$ . Going back to our example of airline seats offered for a specific route, from now on we can think of the product “an airline seat” and a seller “an airline.” Total number of seats of airlines is exogenously given. The airlines simultaneously decide how many of these seats they sell to which customers. The price of the good for the  $k^{th}$  segment is given by:

$$P^k = P(Q^k) \tag{1}$$

where  $q_A^i$  and  $q_B^i$  are the quantities sold in segment  $i$  by  $A$  and  $B$ ;  $Q^k \equiv \sum_{i=1}^k (q_A^i + q_B^i)$  is the total quantity sold in all segments from 1 to  $k$ ; and  $P$  is a twice continuously differentiable, strictly decreasing demand function that represents consumers’ valuations.<sup>8</sup> Moreover, for a given combination of  $Q_A$  and  $Q_B$ , we assume that the revenue functions (which are also the profit functions in our case,

---

<sup>8</sup>The number of segments refers to the number of groups in which the product is sold. It is likely that there will be some leftover consumers that are not served.

given zero costs) of firms  $A$  and  $B$  are strictly concave in  $s_A$  and  $s_B$ , respectively.<sup>9</sup> The optimization problem of firm  $A$  is given by:<sup>10</sup>

$$\begin{aligned} \max \pi_A &= Q_A \sum_{i=1}^K P^i s_A^i \\ \text{s.t. } s_A^i &\geq 0 \text{ and } \sum_{i=1}^K s_A^i = 1 \end{aligned} \quad (2)$$

Lemma 1 and Propositions 1 and 2, which we present below, contribute to the solution of optimization problem (2) above. In Lemma 1 below we show that both firms are active in the top segment and that the segments where a firm sells are consecutive. Also, the bigger firm is active in all  $K$  segments. Hence, there may be segments where the smaller firm is not active. We identify those segments where the smaller firm is not active through an index denoted by  $\hat{i}$ . The segments with index numbers greater than  $\hat{i}$  have only the bigger firm active in them. In top segment(s), except the last segment where the smaller firm is active (i.e., segment  $\hat{i} - 1$ ), firms match their quantities. By top segments we mean the segments with lower indices which represent the higher valuation customers and  $\hat{i}$  is the last segment (from the top) in which the smaller firm is not active. Even though there are  $K$  segments, we are using the index up to  $K + 1$  in order to include the case where  $Q_A = Q_B$  or they are so close that  $A$  is active in all the segments. Hence,  $\hat{i} = K + 1$  means that  $s_A^i > 0$  in all segments,  $i = 1, 2, \dots, K$ .

### Lemma 1

1. Assume that  $Q_A \leq Q_B$  and for some segment  $i \in \{1, 2, \dots, K\}$  we have  $s_A^i = 0$ , then  $s_A^{i+1} = 0$ .
2. Firm  $B$ 's share in  $j$ th segment,  $s_B^j > 0$  for  $j \in \{1, 2, \dots, K\}$ .
3. Let  $\hat{i} \in \{3, \dots, K, K + 1\}$  be such that  $s_A^{\hat{i}} = s_A^{\hat{i}+1} \dots = s_A^{K+1} = 0$  and  $s_A^j > 0$  for  $j < \hat{i}$ . Then for  $j < \hat{i} - 1$  we have  $q_A^j = q_B^j$ .

Now we provide a proposition that describes the behavior of the firms in all segments for a general demand function. Even though we do not have a closed-form solution, this proposition gives a recursive way to get an explicit solution for a specific demand function up to finding  $\hat{i}$ . After the proposition we describe an

---

<sup>9</sup>The demand curve not being too convex is one of the requirements. Notice for example, in the monopoly case with no price discrimination we require the inverse demand function to be "less convex" than  $P(Q) = \frac{1}{Q}$ . See Novshek [13], Roberts and Sonnenschein [16], Szidarowsky and Yakowitz [20], Tirole [22], and Vives [24] for conditions on existence in setups without price discrimination.

<sup>10</sup>Note that the optimization problem for firm  $B$  is exactly the same.

algorithm to find such a solution. Later in this paper, in Corollaries 1, 2, and 3, we give an explicit solution for the linear demand case as an example.

**Proposition 1** *Assume that  $Q_A \leq Q_B$ . Let  $\hat{i} \in \{2, 3, \dots, K, K+1\}$  be such that  $s_A^{\hat{i}} = s_A^{\hat{i}+1} \dots = s_A^K = s_A^{K+1} = 0$  and  $s_A^j > 0$  for  $j < \hat{i}$ . The optimal shares for A and B are described in Algorithm 1.*

**Algorithm 1** *Given the value of  $\hat{i}$ , the algorithm provides the quantity allocations for each segment. Recall that A may not be active for some of the segments. Hence, the algorithm requires a case analysis. There are three cases to consider. Case I considers the segments where both A and B are fully active so that A matches the quantity of B. Case II considers the segment where both A and B are active but where the capacity of A is not large enough to match the quantity of B for that particular segment. Finally, Case III considers the segments where only B is active. Based on these cases, the solution algorithm is given as follows.*

*From the cases below, we can recursively solve  $s_B^j$  in terms of  $s_B^1$  for  $j \leq K$ . Moreover, since we have  $\sum_{k=1}^K s_B^k = 1$ , we can solve for  $s_B^1$ . Once we have the solution for  $s_B$ 's we can solve for  $s_A$ 's as follows. From Case I, we can recursively solve  $s_A^j$  in terms of  $s_A^1$  for  $j < \hat{i} - 1$ . Since we have  $s_A^{\hat{i}-1} = 1 - \sum_{k=1}^{\hat{i}-2} s_A^k$ , we can solve for  $s_A^{\hat{i}-1}$  in terms of  $s_A^1$  as well. In order to solve for  $s_A^1$ , we use Lemma 1. That is, given  $s_B^1$  the solution for  $s_A^1$  is given by:<sup>11</sup>*

$$s_A^1 = \frac{Q_B}{Q_A} s_B^1.$$

*Note that depending on the value of  $\hat{i}$  some of the cases disappear. Hence, the sequence of shares might start from Case II or Case III rather than Case I. Whenever  $\hat{i} > 3$  the solution algorithm starts from Case I; if  $\hat{i} = 3$ , the solution algorithm starts from Case II; and if  $\hat{i} = 2$ , the solution algorithm starts from Case III.*

*Case I ( $j < \hat{i} - 2$ ):*

$$P(2Q_A \sum_{k=1}^j s_A^k) - P(2Q_A \sum_{k=1}^{j+1} s_A^k) = -\frac{\partial P^j}{\partial Q} Q_A s_A^j \quad (3)$$

$$P(2Q_B \sum_{k=1}^j s_B^k) - P(2Q_B \sum_{k=1}^{j+1} s_B^k) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \quad (4)$$

---

<sup>11</sup>Note that this statement holds for the  $\hat{i} > 2$  case. The case where  $\hat{i} = 2$  is trivial as  $s_A^1 = 1$ .



Case II ( $j = \hat{i} - 2$ ):

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - P(Q_A(1 + \sum_{k=1}^{\hat{i}-2} s_A^k) + Q_B s_B^{\hat{i}-1}) = -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_A s_A^{\hat{i}-2} \quad (5)$$

$$P(2Q_B \sum_{k=1}^{\hat{i}-2} s_B^k) - P(Q_A + Q_B \sum_{k=1}^{\hat{i}-1} s_B^k) = -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_B s_B^{\hat{i}-2} \quad (6)$$

Case III ( $j > \hat{i} - 2$ ):

$$s_A^j = 0 \text{ for } j > \hat{i} - 1 \quad (7)$$

$$s_A^{\hat{i}-1} = 1 - \sum_{k=1}^{\hat{i}-2} s_A^k \quad (8)$$

$$P(Q_A + Q_B \sum_{k=1}^j s_B^k) - P(Q_A + Q_B \sum_{k=1}^{j+1} s_B^k) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j \text{ for } j < K \quad (9)$$

$$s_B^K = 1 - \sum_{k=1}^{K-1} s_B^k. \quad (10)$$

In Proposition 1 we described the conditions for equilibrium shares for a general demand function. One particular implication of this proposition is that unless the demand function is “too convex,” the shares of the higher-end segments are greater than those of the lower-end ones. In the following proposition, we give more conditions that will help identify  $\hat{i}$ . The first statement of the proposition along with Lemma 1 shows that there is no segment where the smaller firm puts more quantity than the bigger firm.

**Proposition 2** *The shares of firms in the last segment where A is active, i.e., the segment  $\hat{i} - 1$ , must satisfy:*

$$Q_A s_A^{\hat{i}-1} \leq Q_B s_B^{\hat{i}-1} \text{ for } 2 \leq \hat{i} \leq K + 1 \quad (11)$$

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - P(2Q_A \sum_{k=1}^{\hat{i}-1} s_A^k) \leq -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_A s_A^{\hat{i}-2} \text{ for } 3 \leq \hat{i} \leq K + 1 \quad (12)$$

At this point we would like to mention that the shares that are decided by the above results are invariant to any affine transformation of the demand function. In other words, two demand functions  $P$  and  $\tilde{P}$  where  $\tilde{P} = \alpha + \beta P$  would lead to the same solution for the shares. In what follows we solve the linear demand

case. For a general demand function the equilibrium can be calculated in a similar fashion.

### 3 The Linear Demand Case

In this section we present our main results for the linear demand case. We consider the linear demand given by  $P^j = a - Q^j$ . Using the propositions stated above we find the closed-form solution for the equilibrium, which turns out to be unique.<sup>12</sup> In line with the previous section, in the equilibrium both firms are active in the top segment(s) and the bigger firm is active in all  $K$  segments. The firms match the quantities in the top segments until segment  $\hat{i} - 2$ . Also, in each segment until segment  $\hat{i} - 2$ , the firms put exactly half of the quantity that they put in the previous segment. Starting from segment  $\hat{i} - 1$  the bigger firm splits the quantity equally in all segments. Recall that this behavior of the bigger firm is like that of a monopolist in those segments.

**Corollary 1** *Let  $\hat{i} \in \{2, \dots, K, K + 1\}$  be such that  $s_A^{\hat{i}} = s_A^{\hat{i}+1} \dots = s_A^K = 0$  and  $s_A^j > 0$  for all  $j < \hat{i}$ . Moreover, assume that the demand is linear, given by:*

$$P^j = a - Q^j. \quad (13)$$

*The optimal shares for A and B are described as follows:*

*Case 1 ( $\hat{i} = 2$ ):*

$$\begin{aligned} s_A^1 &= 1 \text{ and } s_A^j = 0 \text{ if } j > 1 \\ s_B^j &= \frac{1}{K} \text{ for } j = 1, 2, \dots, K \end{aligned}$$

*Case 2 ( $\hat{i} \geq 3$ ):*

$$\begin{aligned} s_A^j &= \begin{cases} \frac{1}{2^{j-1}} s_A^1 & \text{if } j < \hat{i} - 1 \\ 1 - (2 - \frac{1}{2^{\hat{i}-3}}) s_A^1 & \text{if } j = \hat{i} - 1 \\ 0 & \text{if } j > \hat{i} - 1 \end{cases} \\ s_B^j &= \begin{cases} \frac{1}{2^{j-1}} s_B^1 & \text{if } j < \hat{i} - 1 \\ 2s_B^1 - \frac{Q_A}{Q_B} & \text{if } j \geq \hat{i} - 1. \end{cases} \end{aligned}$$

---

<sup>12</sup>Note that any linear inverse demand function will lead to exactly the same solution as we have mentioned earlier.

The following corollary states the behavior of firm  $A$  in the last segment(s). In segment  $\hat{i} - 1$  it just puts the remainder, which is no more than half of what he puts in segment  $\hat{i} - 2$ .

**Corollary 2** *For any  $\hat{i} = 3, \dots, K + 1$  we have:*

$$2s_A^{\hat{i}-1} \leq s_A^{\hat{i}-2}. \quad (14)$$

The following corollary, together with Corollary 1, characterizes the solution for general  $\hat{i} \geq 2$ . For a given  $Q_A/Q_B$  ratio,  $\hat{i}$  is unique and so is the equilibrium. When the gap between capacities of firms increases (i.e.,  $Q_A/Q_B$  decreases), the number of segments where the firm with smaller capacity is active (i.e.,  $\hat{i} - 1$ ) would decrease. The firm with smaller capacity would concentrate on the high-end segments. The reason is that the firm with smaller capacity would have a higher shadow cost compared to the other firm, which makes it relatively harder for it to stay in the low-end segments.

**Corollary 3** *The shares for the first segments are given as follows:*

$$\begin{aligned} s_A^1 &= \begin{cases} \frac{Q_B}{Q_A} \frac{1 + \frac{Q_A}{Q_B} K_{\hat{i}}}{\frac{H_{\hat{i}}}{2^{\hat{i}-3}} + 2K_{\hat{i}}} & \text{if } \hat{i} > 2 \\ 1 & \text{if } \hat{i} = 2 \end{cases} \\ s_B^1 &= \begin{cases} \frac{1 + \frac{Q_A}{Q_B} K_{\hat{i}}}{\frac{H_{\hat{i}}}{2^{\hat{i}-3}} + 2K_{\hat{i}}} & \text{if } \hat{i} > 2 \\ 1/K & \text{if } \hat{i} = 2 \end{cases} \end{aligned}$$

where the unique  $\hat{i}$  is characterized by:

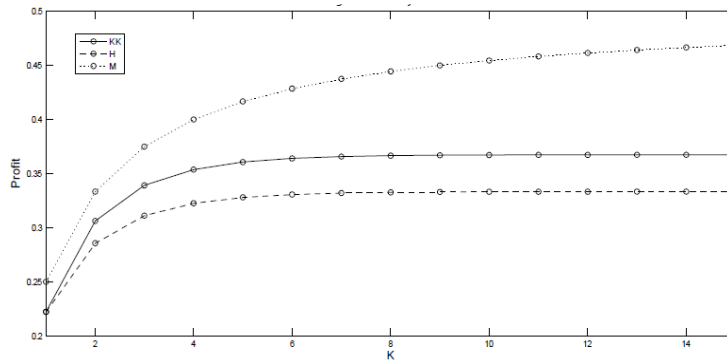
$$\begin{aligned} \frac{1 + 2H_{\hat{i}}}{K_{\hat{i}} + 2H_{\hat{i}}} &\geq \frac{Q_A}{Q_B} > \frac{H_{\hat{i}}}{K_{\hat{i}} + H_{\hat{i}}} \\ H_{\hat{i}} &= 2^{\hat{i}-2} - 1 \\ K_{\hat{i}} &= K - \hat{i} + 2. \end{aligned}$$

Now that we have identified the unique equilibrium of the share allocation game with exogenously given capacities, we explore the equilibria in the games where the capacities themselves are endogenous.

## 4 Generalization of Cournot As an Example

In this section we provide a generalization of the benchmark Cournot competition model and compare it with Hazledine’s [6] price discrimination model. For both settings we assume that there are two firms in the market and the marginal costs are equal to zero. The firms divide the consumers into  $K$  segments according to their reservation prices. The demand is assumed to be linear and given by Equation (13). We set  $a = 1$ . In Hazledine’s [6] framework the firms are playing a one-stage game in which they simultaneously choose the quantities that they assign to each segment. By contrast, the second stage of our two-stage framework corresponds to the share allocation game introduced in the previous section.

A crucial difference in solving the two models is that the profit function of each firm in the first stage of our two-stage model is a piecewise function. Each piece corresponds to a different  $\hat{i}$ , which in turn is determined by the ratio of first stage quantities,  $\frac{Q_A}{Q_B}$ . A difficulty in finding the best response functions of the firms is that the calculations must take into account the various cases corresponding to various  $\hat{i}$ ’s. However, for finding the symmetric equilibrium we only need to consider the piece corresponding to  $\hat{i} = K + 1$ . For the two firms and two prices (segments) case, we find that the equilibrium quantities of each firm is  $9/23$ , in contrast with Hazledine’s [6] corresponding quantity,  $3/7$ . The average price<sup>13</sup> and profits respectively are  $9/23$  and  $81/529$ , as opposed to  $1/3$  and  $1/7$  in Hazledine’s [6] framework. For  $K = 1, 2, \dots, 15$  we plot the profits, quantities, and average prices for comparison. Figures 1-4 compare the symmetric equilibrium outcomes of our two-stage game, the equilibrium outcomes of Hazledine [6], and the equilibrium outcomes of price discriminating monopoly. From now on, we denote our model by “KK,” Hazledine’s [6] by “H,” and monopoly by “M.”



<sup>13</sup>By average price we mean output-weighted average price.

Figure 1: Industry profit

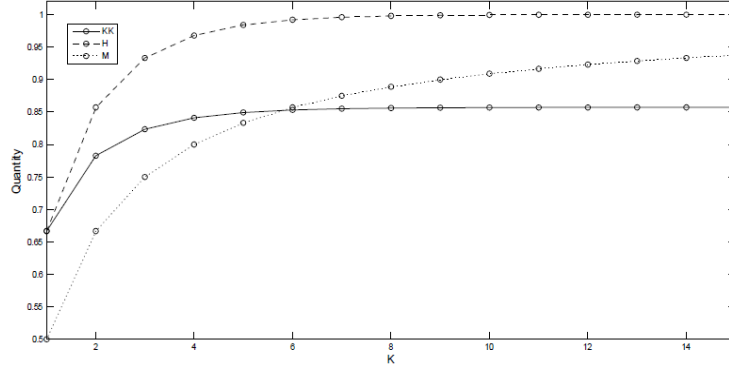


Figure 2: Industry quantity

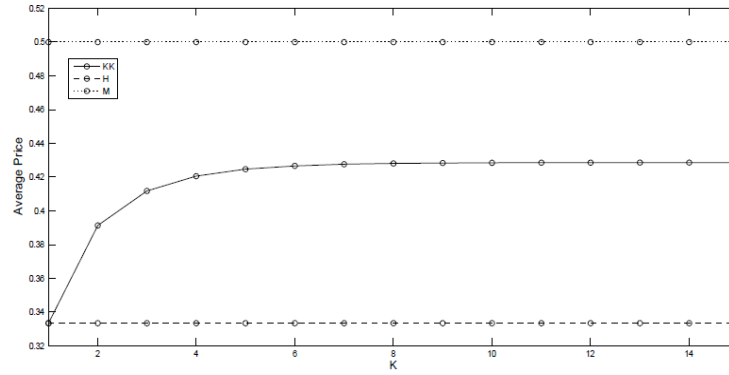


Figure 3: Average price

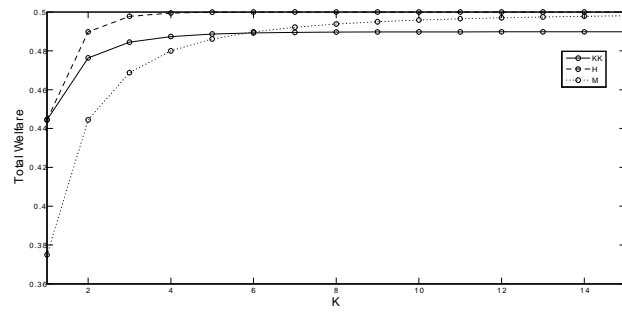


Figure 4: Total welfare

In general, the profits of  $A$  and  $B$  are given by:

$$\begin{aligned}\pi_A &= (1 - f_i Q_i) Q_A + g_i Q_i^2 \\ \pi_B &= Q_B - h_i Q_i^2 + K_i Q_i Q_A - \frac{(K_i - 1) K_i}{2} Q_A^2\end{aligned}$$

where

$$\begin{aligned}
H_{\hat{i}} &= 2^{\hat{i}-2} - 1 \\
K_{\hat{i}} &= K - \hat{i} + 2 \\
Q_{\hat{i}} &= Q_B + K_{\hat{i}} Q_A \\
\frac{1 + 2H_{\hat{i}}}{K_{\hat{i}} + 2H_{\hat{i}}} &\geq \frac{Q_A}{Q_B} > \frac{H_{\hat{i}}}{K_{\hat{i}} + H_{\hat{i}}} \\
x_{\hat{i}} &= \frac{1}{2^{\hat{i}}} \\
f_{\hat{i}} &= \frac{2(1 - 2x_{\hat{i}})}{1 - 4x_{\hat{i}} + K_{\hat{i}}} \\
g_{\hat{i}} &= \frac{2}{3} \frac{8x_{\hat{i}}^2 - 6x_{\hat{i}} + 1}{(1 - 4x_{\hat{i}} + K_{\hat{i}})^2} \\
h_{\hat{i}} &= \frac{8(8x_{\hat{i}}^2 - 6x_{\hat{i}} + 1) + 12K_{\hat{i}}(1 - 2x_{\hat{i}}) + 3K_{\hat{i}}(K_{\hat{i}} - 1)}{6(1 - 4x_{\hat{i}} + K_{\hat{i}})^2}.
\end{aligned}$$

The portion of the best response capacity of firm  $A$  and  $B$  corresponding the  $\hat{i} = K + 1$  is:

$$\begin{aligned}
Q_A &= \frac{3(2^K - 1) - (2^{K+1} - 1) Q_B}{5(2^K) - 4} \\
Q_B &= \frac{3(2^K - 1) - (2^{K+1} - 1) Q_A}{5(2^K) - 4}.
\end{aligned}$$

Therefore, in the symmetric equilibrium the profit, quantity, and average price are given by:<sup>14</sup>

$$\begin{aligned}
\pi_A &= \left( \frac{3(2^K - 1)}{7(2^K) - 5} \right)^2 \\
Q_A &= \frac{3(2^K - 1)}{7(2^K) - 5} \\
\frac{\pi_A}{Q_A} &= \frac{3(2^K - 1)}{7(2^K) - 5}.
\end{aligned}$$

---

<sup>14</sup>We don't provide the relevant values for  $B$  as they would be the same.

Corresponding equilibrium values for Hazledine [6] are given by:

$$\begin{aligned}\pi_A^H &= \frac{2^K - 1}{6(2^K) - 3} \\ Q_A^H &= \frac{2^K - 1}{2^{K+1} - 1} \\ \frac{\pi_A^H}{Q_A^H} &= \frac{1}{3}.\end{aligned}$$

In the limit when  $K \rightarrow \infty$  the equilibrium values become  $\pi_A = \frac{9}{49}$  and  $Q_A = \frac{\pi_A}{Q_A} = \frac{3}{7}$ . Hence, in the presence of competition even if the firms can charge many prices there is deadweight loss. This contrasts with both Hazledine [6] and the price discriminating monopoly case where for large  $K$  there is no deadweight loss. A monopolist can set quantities without being distracted by the effects of competition. However, in the price discrimination framework, depending on the setting (Hazledine [6] versus capacity choice price discrimination game (KK)), competition can have a negative effect on total welfare. Hazledine [7] argues that Air New Zealand and Qantas use  $K = 12$ . If other airlines use a similar number of segments and play KK, then an antitrust authority valuing the total welfare might have higher incentives to approve mergers in the airline industry compared to industries without price discrimination. Hence, the required efficiency gain for such a merger is potentially lower. If the antitrust authority ignores this, then this might lead to an over-rejection of mergers. KK dominates its one-stage counterpart in terms of profits. Conditional on the second stage outcomes, the firms can coordinate their capacities (total quantities) better compared with Hazledine's [6] counterpart. This results in lower quantities and higher profits for the firms. One implication of this is that firms can potentially benefit from the presence of capacity constraints. In Hazledine's [6] setting the average price is invariant to the number of segments,  $K$ . This invariance result is shown to be robust to other settings. For example, Bakó and Kálec-Simon [2] show the invariance result for the asymmetric cost case and Kutlu [11] shows the invariance result for a functional form which is nesting constant elasticity demand function. However, in our capacity choice framework this invariance result breaks down.

## 5 Conclusion

We studied the price discrimination and imperfect competition in a homogeneous goods framework. We modelled this as a two-stage game as opposed to the existing

works in this particular framework. One of our main findings is that deadweight loss exists for all levels of price discrimination. Hence, if the number of price segments is large, the monopoly may be socially preferred over a duopoly. This contrasts with the price discriminating monopolist and one-stage price discriminating duopoly. Therefore, a welfarist social planner should consider protecting monopolies or approving mergers when evidence of two stage price discrimination is found. Another finding is that the output-weighted average price will increase as the level of price discrimination increases. This contrasts with the earlier findings that suggest that output-weighted average price is invariant to the level of price discrimination and this invariance is robust. Thus it has been suggested that the average price can be used for estimation purposes without worrying about price discrimination. Our result changes this perception and indicates that caution must be taken if there is evidence of price discrimination.

A generalization of our model to the many firms case can also be used in merger analysis in the price discrimination framework where merging is a way to expand the capacities of firms. Salant, Switzer, and Reynolds [17] show that, in general, without a form of cost reduction the mergers are not profitable in the Cournot framework. One way to deal with this problem is to allow capacity expansions ex-post the merger. Of course, in practice the merger analysis is more complicated than this. A better model for analyzing the effects of mergers would also incorporate other factors, such as the dynamic factors and efficiency.<sup>15</sup>

## 6 Appendix: Proofs

### 6.1 Proof of Lemma 1

The Lagrangian for the optimization problem (2) is given by:

$$\mathcal{L}_A = \pi_A + \mu_A \left( \sum_{j=1}^K s_A^j - 1 \right) \quad (15)$$

Let  $\tilde{\mu}_A = \frac{\mu_A}{Q_A}$ .<sup>16</sup> For any  $i = 1, 2, \dots, K$  the Kuhn-Tucker conditions are given

---

<sup>15</sup>See Kutlu and Sickles [12] for a dynamic model considering the efficiencies of firms when measuring their market powers.

<sup>16</sup>Note that we are solving the problem of an active firm. Therefore it is assumed that  $Q_A > 0$ .



by:<sup>17</sup>

$$\begin{aligned}
P^i + A_i + \tilde{\mu}_A &\leq 0 \\
(P^i + A_i + \tilde{\mu}_A)s_A^k &= 0 \\
\sum_{k=1}^K s_A^k &= 1 \\
s_A^i &\geq 0
\end{aligned}$$

where  $A_i = \sum_{k=i}^K \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_A^k} s_A^k$ . In what follows we assume that  $A_i = \sum_{k=i}^K \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_A^k} s_A^k$  and  $B_i = \sum_{k=i}^K \frac{\partial P^k}{\partial Q} \frac{\partial Q^k}{\partial s_B^k} s_B^k$  for the sake of notational simplicity.

Now, we prove the statement 1 of Lemma 1. Let us assume, to get a contradiction, that  $s_A^i = 0$  and  $s_A^{i+1} > 0$  for some  $i \in \{1, 2, \dots, K-1\}$ . Then we have:

$$P^i \leq -A_i - \tilde{\mu}_A = -A_{i+1} - \tilde{\mu}_A = P^{i+1}$$

Here the inequality comes from the Kuhn-Tucker conditions; the first equality follows from our assumption that  $s_A^i = 0$ ; the second equality follows from the Kuhn-Tucker conditions given that  $s_A^{i+1} > 0$ . Hence,  $P^i \leq P^{i+1}$ . But by the monotonicity of the demand  $P^i \geq P^{i+1}$ , implying that  $P^i = P^{i+1}$ . This in turn implies that there are  $K-1$  segments, which is a contradiction.

Now, we prove the statement 2 of Lemma 1. Let  $\hat{i} \in \{2, \dots, K, K+1\}$  be such that  $s_A^{\hat{i}} = s_A^{\hat{i}+1} \dots = s_A^{K+1} = 0$  and  $s_A^j > 0$  for all  $j < \hat{i}$ . In order to prove the lemma, we consider two cases.

Case 1 ( $\hat{i} \leq K$ ): If  $\hat{i} \leq K$ , then  $s_B^K > 0$ . Otherwise, there will not be  $K$  segments which is a contradiction.

Case 2 ( $\hat{i} = K+1$ ): The arguments from statement 1 of Lemma 1 holds for  $B$  as well. Therefore, it is clear that  $s_B^1 > 0$ . Otherwise,  $s_B^2 = \dots = s_B^K = 0$ , implying that  $Q_B = 0$ , which is a contradiction. Let's assume that  $s_B^j > 0$  for all  $j < t$ . We will show that  $s_B^t > 0$ . Assume this is not the case, i.e.,  $s_B^t = s_B^{t+1} = \dots = s_B^K = 0$ . From the Kuhn-Tucker conditions we know that:

$$P^j + A_j + \tilde{\mu}_A = 0 \tag{16}$$

$$P^j + B_j + \tilde{\mu}_B = 0 \tag{17}$$

$$P^t + A_t + \tilde{\mu}_A = 0 \tag{18}$$

$$P^t + B_t + \tilde{\mu}_B \leq 0 \tag{19}$$

---

<sup>17</sup>For notational simplicity we represent  $\frac{\partial P(Q^j)}{\partial Q}$  by  $\frac{\partial P^j}{\partial Q}$ .

Subtracting the equality (18) from the inequality (19) gives:

$$B_t - A_t + \tilde{\mu}_B - \tilde{\mu}_A \leq 0$$

From (16) and (17) we know that:

$$\tilde{\mu}_B - \tilde{\mu}_A = A_j - B_j$$

Therefore, we have:

$$B_t - A_t + A_j - B_j \leq 0 \quad (20)$$

$$B_j - A_j + A_{j-1} - B_{j-1} = 0 \quad (21)$$

From equations (20) and (21), we have:

$$\begin{aligned} -Q_B \frac{\partial P^{t-1}}{\partial Q} s_B^{t-1} + Q_A \frac{\partial P^{t-1}}{\partial Q} s_A^{t-1} &\leq 0 \\ -Q_B \frac{\partial P^{t-2}}{\partial Q} s_B^{t-2} + Q_A \frac{\partial P^{t-2}}{\partial Q} s_A^{t-2} &= 0 \\ &\vdots \\ -Q_B \frac{\partial P^1}{\partial Q} s_B^1 + Q_A \frac{\partial P^1}{\partial Q} s_A^1 &= 0 \end{aligned}$$

From monotonicity of demand, we have  $\frac{\partial P^j}{\partial Q} < 0$ . Therefore:

$$Q_B s_B^{t-1} \leq Q_A s_A^{t-1}$$

Summing over segments  $1, 2, \dots, t-1$  we get:

$$Q_B \sum_{k=1}^{t-1} s_B^k \leq Q_A \sum_{k=1}^{t-1} s_A^k$$

or

$$Q_B \leq Q_A \sum_{k=1}^{t-1} s_A^k < Q_A$$

The strict inequality follows from the fact that  $A$  is active in all segments until segment  $K$ . This is a contradiction.

Finally, we prove statement 3 of Lemma 1. Note that by statement 2 of Lemma 1 we have  $s_B^j > 0$  for  $j \in \{1, 2, \dots, K\}$ . Hence, for all  $j < \hat{i}$  we have  $P^j = -A_j - \tilde{\mu}_A = -B_j - \tilde{\mu}_B$ . Hence,  $P^j - P^{j+1} = q_A^j = q_B^j$ .

■

## 6.2 Proof of Proposition 1

Note that for  $j < \hat{i} - 1$  we have:

$$\begin{aligned} P(Q^j) - P(Q^{j+1}) &= (-A_j - \tilde{\mu}_A) - (-A_{j+1} - \tilde{\mu}_A) \\ &= -\frac{\partial P^j}{\partial Q} Q_A s_A^j \end{aligned}$$

Also, by statement 2 of Lemma 1, using the similar steps as above, we get:

$$P(Q^j) - P(Q^{j+1}) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j$$

For Case I, we have  $j < \hat{i} - 2$ . Therefore  $Q^j = Q_A \sum_{k=1}^j s_A^k + Q_B \sum_{k=1}^j s_B^k$ . Since  $j < \hat{i} - 2$ , by Lemma 1 we have  $Q_A s_A^j = Q_B s_B^j$ . Hence,  $Q^j = 2Q_A \sum_{k=1}^j s_A^k = 2Q_B \sum_{k=1}^j s_B^k$  and  $Q^{j+1} = 2Q_A \sum_{k=1}^{j+1} s_A^k = 2Q_B \sum_{k=1}^{j+1} s_B^k$ .

For Case II, we have  $j = \hat{i} - 2$ . Therefore  $Q^j = 2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k = 2Q_B \sum_{k=1}^{\hat{i}-2} s_B^k$  and  $Q^{j+1} = Q_A(1 + \sum_{k=1}^{\hat{i}-2} s_A^k) + Q_B s_B^{\hat{i}-1} = Q_A + Q_B \sum_{k=1}^{\hat{i}-1} s_B^k$ .

For Case III, notice that  $Q_A$  is exhausted after segment  $\hat{i} - 1$ . For segment  $\hat{i} - 1$ ,  $s_A^{\hat{i}-1}$  is the residual share for  $A$ . By statement 2 of Lemma 1,  $B$  is active in segments  $\hat{i}, \hat{i} + 1, \dots, K$ , i.e.,  $s_B^{\hat{i}}, s_B^{\hat{i}+1}, \dots, s_B^K > 0$ . Therefore from the Kuhn-Tucker conditions for all  $j = \hat{i}, \hat{i} + 1, \dots, K - 1$  we have:

$$\begin{aligned} P^j + B_j + \tilde{\mu}_B &= 0 \\ P^{j+1} + B_{j+1} + \tilde{\mu}_B &= 0 \end{aligned}$$

Hence, we have:

$$P^j - P^{j+1} = -\frac{\partial P^j}{\partial Q} Q_B s_B^j$$

or

$$P(Q_B \sum_{k=1}^j s_B^k + Q_A) - P(Q_B \sum_{k=1}^{j+1} s_B^k + Q_A) = -\frac{\partial P^j}{\partial Q} Q_B s_B^j$$

■

### 6.3 Proof of Proposition 2

First, we prove the inequality (11). From the Kuhn-Tucker conditions we know that:

$$\begin{aligned} P^{\hat{i}} + A_i + \tilde{\mu}_A &\leq 0 \\ P^{\hat{i}} + B_i + \tilde{\mu}_B &= 0 \\ P^{\hat{i}-1} + A_{i-1} + \tilde{\mu}_A &= 0 \\ P^{\hat{i}-1} + B_{i-1} + \tilde{\mu}_B &= 0 \end{aligned}$$

Then we have:

$$\begin{aligned} P^{\hat{i}} - P^{\hat{i}-1} + A_i - A_{i-1} &\leq 0 \\ P^{\hat{i}} - P^{\hat{i}-1} + B_i - B_{i-1} &= 0. \end{aligned}$$

Hence:

$$A_i - A_{i-1} \leq B_i - B_{i-1}$$

or

$$-\frac{\partial P^{\hat{i}-1}}{\partial Q} Q_A s_A^{\hat{i}-1} \leq -\frac{\partial P^{\hat{i}-1}}{\partial Q} Q_B s_B^{\hat{i}-1}.$$

By monotonicity of the demand we know that  $\frac{\partial P^{\hat{i}-1}}{\partial Q} < 0$ . Therefore we have:

$$Q_A s_A^{\hat{i}-1} \leq Q_B s_B^{\hat{i}-1}$$

Now, we prove the inequality (12). From Proposition 1 we know that:

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - P(Q_A(1 + \sum_{k=1}^{\hat{i}-2} s_A^k) + Q_B s_B^{\hat{i}-1}) = -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_A s_A^{\hat{i}-2}$$

or

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k + Q_A s_A^{\hat{i}-1} + Q_B s_B^{\hat{i}-1}) = -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_A s_A^{\hat{i}-2}.$$

Since  $Q_A s_A^{\hat{i}-1} \leq Q_B s_B^{\hat{i}-1}$  by monotonicity of the demand we have:

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k + Q_A s_A^{\hat{i}-1} + Q_B s_B^{\hat{i}-1}) \leq P(2Q_A \sum_{k=1}^{\hat{i}-1} s_A^k).$$

Therefore:

$$P(2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - P(2Q_A \sum_{k=1}^{\hat{i}-1} s_A^k) \leq -\frac{\partial P^{\hat{i}-2}}{\partial Q} Q_A s_A^{\hat{i}-2}.$$

■

## 6.4 Proof of Corollary 1

For Case 1, note that by definition of  $\hat{i}$  and statement 2 of Lemma 1 we have  $s_A^1 = 1$ . From Equation (9) and Equation (10) we have:

$$\begin{aligned} (a - Q_A - Q_B \sum_{k=1}^j s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{j+1} s_B^k) &= Q_B s_B^j \text{ for } j < K \\ s_B^K &= 1 - \sum_{k=1}^{K-1} s_B^k. \end{aligned}$$

Hence:

$$\begin{aligned} s_B^{j+1} &= s_B^j \text{ for } j < K \\ s_B^K &= 1 - \sum_{k=1}^{K-1} s_B^k. \end{aligned}$$

This implies that:

$$s_B^j = \frac{1}{K}.$$

For Case 2, we only prove the  $\hat{i} > 3$  case. The  $\hat{i} = 3$  case is similar. For  $j < \hat{i} - 1$ , by Equation (3) and Equation (4), for any  $j < \hat{i} - 2$  we have:

$$\begin{aligned} (a - 2Q_A \sum_{k=1}^j s_A^k) - (a - 2Q_A \sum_{k=1}^{j+1} s_A^k) &= Q_A s_A^j \\ (a - 2Q_B \sum_{k=1}^j s_B^k) - (a - 2Q_B \sum_{k=1}^{j+1} s_B^k) &= Q_B s_B^j \end{aligned}$$

Hence:

$$\begin{aligned} 2s_A^{j+1} &= s_A^j \\ 2s_B^{j+1} &= s_B^j \end{aligned}$$

Hence, for any  $j < \hat{i} - 1$  we have:

$$\begin{aligned} s_A^j &= \frac{1}{2^{j-1}} s_A^1 \\ s_B^j &= \frac{1}{2^{j-1}} s_B^1. \end{aligned} \tag{22}$$

By Equation (22) and the fact that  $s_A^{\hat{i}-1} = 1 - \sum_{k=1}^{\hat{i}-2} s_A^k$  we have  $s_A^{\hat{i}-1} = 1 - (2 - \frac{1}{2^{\hat{i}-3}}) s_A^1$ . Now, we find  $s_B^{\hat{i}-1}$ . From Equation (6) we know that:

$$\begin{aligned} (a - 2Q_B \sum_{k=1}^{\hat{i}-2} s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{\hat{i}-1} s_B^k) &= Q_B s_B^{\hat{i}-2} \\ s_B^K &= 1 - \sum_{k=1}^{K-1} s_B^k. \end{aligned}$$

Hence:

$$(-2Q_B \sum_{k=1}^{\hat{i}-2} s_B^k) - (a - Q_A - Q_B \sum_{k=1}^{\hat{i}-1} s_B^k) = Q_B s_B^{\hat{i}-2}.$$

Hence, we have:

$$\begin{aligned} Q_A + Q_B s_B^{\hat{i}-1} &= Q_B s_B^{\hat{i}-2} + Q_B \sum_{k=1}^{\hat{i}-2} s_B^k \\ Q_A + Q_B s_B^{\hat{i}-1} &= Q_B s_B^{\hat{i}-3} + Q_B \sum_{k=1}^{\hat{i}-3} s_B^k \\ &\vdots \\ Q_A + Q_B s_B^{\hat{i}-1} &= 2Q_B s_B^1. \end{aligned}$$

This implies that:

$$s_B^{\hat{i}-1} = 2s_B^1 - \frac{Q_A}{Q_B}. \tag{23}$$

The case  $j > \hat{i} - 1$  directly follows from equations (7), (9), and (23). ■

## 6.5 Proof of Corollary 2

By Proposition 2 we know that:

$$(a - 2Q_A \sum_{k=1}^{\hat{i}-2} s_A^k) - (a - 2Q_A \sum_{k=1}^{\hat{i}-1} s_A^k) \leq Q_A s_A^{\hat{i}-2}.$$

Hence:

$$2s_A^{\hat{i}-1} \leq s_A^{\hat{i}-2}.$$

■

## 6.6 Proof of Corollary 3

First, assume that  $\hat{i} > 2$ . Using Case II in Proposition 1 we have:

$$Q_A s_A^{\hat{i}-1} + Q_B s_B^{\hat{i}-1} = Q_A s_A^{\hat{i}-2}. \quad (24)$$

Also from Corollary 2 we have:

$$s_A^{\hat{i}-1} \leq \frac{1}{2} s_A^{\hat{i}-2}.$$

Assume that the quantities that firm  $A$  puts in the segments  $\hat{i} - 1$  and  $\hat{i} - 2$  are  $y$  and  $x$ , respectively. Then, from equation (24), Corollary 1, and Corollary 2 we get the following system, which will characterize  $\hat{i}$ :

$$\begin{aligned} y + x + 2x + 4x + \dots + 2^{\hat{i}-3}x &= Q_A \\ (K - \hat{i} + 2)(x - y) + x + 2x + 4x + \dots + 2^{\hat{i}-3}x &= Q_B \\ 0 &\leq y \leq \frac{x}{2} \\ Q_A &\leq Q_B. \end{aligned}$$

Letting  $H_{\hat{i}} = 2^{\hat{i}-2} - 1$  and  $K_{\hat{i}} = K - \hat{i} + 2$  we have:

$$\begin{aligned} y + H_{\hat{i}}x &= Q_A \\ -K_{\hat{i}}y + (K_{\hat{i}} + H_{\hat{i}})x &= Q_B \\ 0 &< y \leq \frac{x}{2} \\ Q_A &\leq Q_B. \end{aligned} \quad (25)$$

Solving for  $x$  and  $y$  we have:

$$\begin{aligned} x &= \frac{K_{\hat{i}}Q_A + Q_B}{K_{\hat{i}}H_{\hat{i}} + K_{\hat{i}} + H_{\hat{i}}} \\ y &= \frac{K_{\hat{i}}Q_A + H_{\hat{i}}(Q_A - Q_B)}{K_{\hat{i}}H_{\hat{i}} + K_{\hat{i}} + H_{\hat{i}}} \end{aligned}$$

From the inequality (25) we have:

$$\frac{1 + 2H_i}{K_i + 2H_i} \geq \frac{Q_A}{Q_B} > \frac{H_i}{K_i + H_i}. \quad (26)$$

Now, assume that  $\hat{i} = 2$ . Then, by Proposition 2 and Corollary 1 we have:

$$Q_A \leq \frac{1}{K} Q_B.$$

Note that  $H_2 = 0$  and  $K_i = K$ . Hence, the system (26) holds for  $\hat{i} = 2$  as well. Let  $\theta_i = \frac{1+2H_i}{K_i+2H_i}$  and  $\lambda_i = \frac{H_i}{K_i+H_i}$ . Now, we show the uniqueness of the equilibrium. Note that it is enough to show that  $\{(\lambda_i, \theta_i)\}_i$  partitions  $(0, 1]$ . This simply means that for any given  $\frac{Q_A}{Q_B}$  value, there will be one and only one corresponding set  $(\lambda_i, \theta_i]$ . This set identifies the  $\hat{i}$  that gives the equilibrium. First, note that  $\frac{\partial \theta_i}{\partial i} \geq 0$  and  $\frac{\partial \lambda_i}{\partial i} \geq 0$ . Moreover, we know that  $\theta_{K+1} = 1$  and  $\lambda_2 = 0$ . Hence, if  $\lambda_i = \theta_{i-1}$  for any  $i = 3, \dots, K$ , then  $\{(\lambda_i, \theta_i)\}_i$  partitions  $[0, 1]$ . We want to show that:

$$\theta_{i-1} = \frac{1 + 2H_{i-1}}{K_{i-1} + 2H_{i-1}} = \frac{H_i}{K_i + H_i} = \lambda_i$$

or

$$\begin{aligned} (1 + 2H_{i-1})(K_i + H_i) - (K_{i-1} + 2H_{i-1})H_i &= 0 \\ K_i + 2K_iH_{i-1} + H_i + 2H_iH_{i-1} - K_{i-1}H_i - 2H_iH_{i-1} &= 0 \\ K_i + 2K_iH_{i-1} + H_i - K_{i-1}H_i &= 0 \\ K_i + (H_i - 1)K_i + H_i - K_{i-1}H_i &= 0 \\ H_i(K_i + 1) - K_{i-1}H_i &= 0 \\ H_i(K - \hat{i} + 2 + 1) - (K - (\hat{i} - 1) + 2)H_i &= 0 \\ 0 &= 0 \end{aligned}$$

We conclude that for any given  $\frac{Q_A}{Q_B}$  there exists a unique equilibrium for the quantity choices of  $A$  and  $B$ . The equilibrium is determined by the conditions from Corollary 1 and inequality system (26). Finally, the shares for the first



segments are given as follows:

$$\begin{aligned} s_A^1 &= \frac{Q_B}{Q_A} \frac{1 + \frac{Q_A}{Q_B} K_i}{\frac{H_i}{2^{i-3}} + 2K_i} \\ s_B^1 &= \frac{1 + \frac{Q_A}{Q_B} K_i}{\frac{H_i}{2^{i-3}} + 2K_i}. \end{aligned} \tag{27}$$

■

## References

- [1]Armstrong, M. (2006), Recent Developments in the Economics of Price Discrimination, *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.
- [2]Bakó, B. and Kálec-Simon, A. (2012), Price Discrimination in Asymmetric Cournot Oligopoly, *Economics Letters*, 116, 301-303.
- [3]Borenstein, S. (1985), Price Discrimination in Free-Entry Markets, *RAND Journal of Economics*, 16, 380-397.
- [4]Chen, Y. (1999), Oligopoly Price Discrimination and Resale Price Maintenance, *RAND Journal of Economics*, 30, 441-55.
- [5]Formby, J.P. and Millner, E.L. (1989), Output and Welfare Effects of Optimal Price Discrimination in Markets Segmented at the Initiative of the Seller, *European Economic Review*, 33, 1175-1181.
- [6]Hazledine, T. (2006), Price Discrimination in Cournot-Nash Oligopoly, *Economics Letters*, 93, 413-420.
- [7]Hazledine, T. (2010), Oligopoly Price Discrimination with Many Prices, *Economics Letters*, 109, 150-153.
- [8]Holmes, T.J. (1989), The Effects of Third-Degree Price Discrimination in Oligopoly, *American Economic Review*, 79, 244-250.
- [9]Kreps, D.M. and Scheinkman, J. (1983), Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, *Bell Journal of Economics*, 14, 326-337.

- [10]Kutlu, L. (2009), Price Discrimination in Stackelberg Competition, *Journal of Industrial Economics*, 57, 364.
- [11]Kutlu, L. (2012), Price Discrimination in Cournot Competition, *Economics Letters*, 117, 540-543.
- [12]Kutlu, L. and Sickles, R.C. (2012), Estimation of Market Power in the Presence of Firm Level Inefficiencies, *Journal of Econometrics: The Econometrics of Auctions and Games*, 168, 141-155.
- [13]Novshek, W. (1985), On the Existence of Cournot Equilibrium, *Review of Economic Studies*, 52, 85-98.
- [14]Renault, R. and Anderson, S.P. (2011), Price Discrimination, Handbook in Transport Economics Edward Elgar, De Palma, A., R. Lindsey, E. Quinet and R. Vickerman (eds.).
- [15]Robinson, J. (1933), Economics of Imperfect Competition, *London: Macmillan*.
- [16]Roberts, J. and Sonnenschein, H. (1976), On the Existence of Cournot Equilibrium without Concave Profit Function, *Journal of Economic Theory*, 13, 112-117.
- [17]Salant, S.W., Switzer, S., and Reynolds, R.J. (1983), Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium, *Quarterly Journal of Economics*, 98, 185-199.
- [18]Stackelberg, H. F. (1934), Market Structure and Equilibrium (Marktform und Gleichgewicht), Springer-Verlag, Berlin.
- [19]Stole, L.A. (2007), Price Discrimination and Competition, M. Armstrong and R. Porter (ed.), *Handbook of Industrial Organization*, vol. 3, Elsevier, Chapter 34, 2221-2299.
- [20]Szidarowsky, F. and Yakowitz, S. (1977), A New Proof of the Existence and Uniqueness of the Cournot Equilibrium, *International Economic Review*, 18, 787-789.
- [21]Thisse, J.F. and Vives, X. (1988), On the Strategic Choice of Spatial Price Policy, *American Economic Review*, 78, 122-137.

- [22]Tirole, J. (1988), The Theory of Industrial Organization, *Cambridge, MA: M.I.T. Press.*
- [23]Varian, H.R. (1985), Price Discrimination and Social Welfare, *The American Economic Review*, 75, 870-875.
- [24]Vives, X. (1985), Nash Equilibrium in Oligopoly Games with Monotone Best Responses, *CARESS W.P., 85-10, University of Pennsylvania.*